HYPERSONIC THREE-DIMENSIONAL RADIATING GAS FLOW AROUND A WING

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It is known (for instance, [1]) that radiation can substantially influence the gas flow around a body upon reentry into the atmosphere at velocities on the order of the second cosmic and higher, and the radiation heat fluxes are commensurate with the convective or even higher. Theoretical investigations of hypersonic radiating gas flow around bodies were performed mainly numerically (see the bibliography in [1]). Hypersonic flow around a wedge and cone with radiation taken into account (plane and axisymmetric flows, respectively) is examined in [2] in a zeroth approximation of the method of the thin shock layer [3]. In addition, it is important to investigate the features of radiating gas flow around three-dimensional bodies.

In this paper the three-dimensional nonstationary hypersonic flow of a radiating gas in the shock layer near the windward surface of a small span wing is investigated with a surface shape varying in time. Application of the method of the thin shock layer [3, 4] permitted obtaining a general solution of the gasdynamic equations that expresses all the flow parameters in terms of the bow shock shape. The problem for its determination is formulated. A class of exact solutions is obtained. The influence of radiation on the shock layer thickness, the density, the temperature, the pressure distribution is studied. The radiation heat flux distribution to the wing is computed.

1. Let us examine the three-dimensional hypersonic flow around a wing at a finite angle of attack α with radiation taken into account at high temperature. We shall consider the compressed gas layer abutting the windward surface of the wing to be optically transparent, i.e., the mean free path length of the radiation L_r is much less than the characteristic thickness of the compressed layer d (the optical thickness of the layer is $r = d/L_r \ll 1$).

We here neglect radiation absorption in the gas, as is justified for not too low flight altitudes [1]. The state of the gas before and after the bow shock is taken at equilibrium. We write the system of nonstationary motion equations for a radiating gas in the form [1]

$$\begin{gathered} d\mathbf{V}/dt \equiv \partial\mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V} = -(1/\rho)\nabla p, \\ \partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) = 0,\ \rho dh/dt - dp/dt + 4\pi k_p B = 0, \\ p/\rho h = (\varkappa_* - 1)/\varkappa_*,\ \varkappa_*(p,h) = h/e(p,h), \\ p\mu = \rho RT,\ B = (\sigma/\pi)T^4, \end{gathered} \tag{1.1}$$

where x, y, z are Cartesian coordinates in a system coupled to the wing (Fig. 1), t is the time, V = (u, v, w) is the velocity vector, p, ρ, h, e, T, μ are the pressure, density, enthalpy, specific internal energy, temperature, and molecular weight of the gas, $\varkappa_*$ is the effective adiabatic index, R is the universal gas constant, σ is the Stefan—Boltzmann constant, and the expression for the divergence of the radiant energy flux vector is written by using the Planck mean absorption coefficient $k_p(p, T)$ [1]

$$k_pB = \int_0^\infty k_\nu B_\nu d\nu,$$

where B_ν, B are, respectively, the intensity of equilibrium radiation with frequency ν and the total radiation intensity. According to [2], at temperatures $T \lesssim 14{,}000$°K and pressures $P \lesssim 10^5$ Pa we have the approximate analytic dependence

$$k_p(p, T) = apT^n,$$

where a, n are constants. As usual, the radiation energy pressure and density were not taken into account in writing (1.1).

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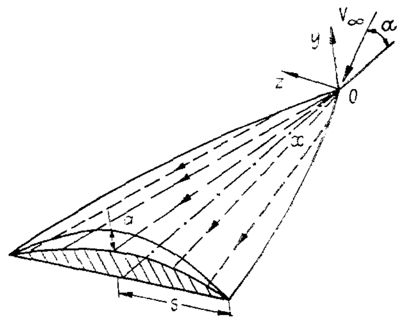

Fig. 1

In the general case the wing has a time-varying shape. The condition of nonpenetration on a wing surface moving at a velocity D_b along the external normal n_b has the form

$$(\mathbf{V}_b - \mathbf{D}_b)\cdot\mathbf{n}_b = 0. \tag{1.2}$$

Values of the functions directly behind the shock moving at a velocity D_s in the direction of the external normal $\mathbf{n}_s$ are related to the unperturbed stream parameters (with the subscript ∞) by the following:

$$\mathbf{V}_s = \mathbf{V}_\infty + V_{n\infty}(1-k)\mathbf{n}_s, \tag{1.3}$$
$$p_s = p_\infty + \rho_\infty V_{n\infty}^2(1-k),\ h_s = h_\infty + \frac{1}{2}V_{n\infty}^2(1-k^2),$$

where $V_{n\infty} = (\mathbf{D}_s - \mathbf{V}_\infty)\cdot\mathbf{n}_s$; $k = \rho_\infty/\rho_s$

2. To investigate the stationary flow around the windward surface of a thin small-span wing of variable shape at large values of the Mach number M_∞ and values of $\varkappa_*$, close to one for the effective adiabatic index behind the shock, we use the method of the thin shock layer [3]. We introduce the small parameter ε, equal to the ratio of the densities on a strong shock ($M_\infty^2\sin^2\alpha >> 1$)

$$\varepsilon = \frac{\varkappa^0 - 1}{\varkappa^0 + 1} \ll 1,\ \varkappa^0 = \varkappa_*\left(\rho_\infty V_\infty^2\sin^2\alpha,\ \frac{1}{2}V_\infty^2\sin^2\alpha\right).$$

The order of magnitude of the temperature ratio on a shock is determined by the product $A = \varepsilon M_\infty^2\sin^2\alpha$. In case $A = O(1)$ as $\varepsilon \to 0$, $M_\infty \to \infty$ [4], the gas temperature in the shock layer is insufficiently high and the influence of radiation is slight. For $A >> 1$, as will be seen later, the temperature can reach values for which the influence of radiation on the aerodynamic characteristics will be significant. Precisely this case is examined below.

Assuming that as $\varepsilon \to 0$, $A \to \infty$, $r = O(\varepsilon A^n) \to 0$ the wing thickness measured from the plane $y = 0$ is of the order of the thickness of the compressed layer ($L\varepsilon\tan\alpha$), while the wing span is of the order of the Mach angle in the compressed layer ($\varepsilon^{1/2}\tan\alpha$) [4], we introduce dimensionless independent variables of the order of one

$$t^0 = tV_\infty\cos\alpha/L,\ x^0 = x/L,\ y^0 = y/L\varepsilon\,\mathrm{tg}\,\alpha,\ z^0 = z/L\varepsilon^{1/2}\,\mathrm{tg}\,\alpha, \tag{2.1}$$

L is the characteristic longitudinal dimension. For flow around a wing with a shock attached to the leading edge, or at least to the wing apex, the desired gasdynamic functions are representable in the form of the expansions

$$u/V_\infty = \cos\alpha + \varepsilon\sin\alpha\,\mathrm{tg}\,\alpha\,u^0(x^0, y^0, z^0, t^0) + \ldots, \tag{2.2}$$
$$v/V_\infty = \varepsilon\sin\alpha v^0(x^0, y^0, z^0, t^0) + \ldots,\quad w/V_\infty = \varepsilon^{1/2}\sin\alpha w^0(x^0, y^0, z^0, t^0) + \ldots,$$
$$p = p_\infty + \rho_\infty V_\infty^2\sin^2\alpha[1 + \varepsilon p^0(x^0, y^0, z^0, t^0) + \ldots],$$
$$\rho/\rho_\infty = \varepsilon^{-1}\rho^0(x^0, y^0, z^0, t^0) + \ldots,\quad 2h/V_\infty^2 = \sin^2\alpha h^0(x^0, y^0, z^0, t^0) + \ldots,$$
$$T/T_\infty = AT^0(x^0, y^0, z^0, t^0) + \ldots,\quad B = (\sigma/\pi)T_\infty^4A^4B^0(x^0, y^0, z^0, t^0) + \ldots,$$
$$k_pL = r\varepsilon^{-1}\,\mathrm{ctg}\,\alpha\,(T^0)^n + \ldots,\quad \mu/\mu_\infty = \mu^0 + \ldots,\quad \varkappa_* = \varkappa^0 + \ldots$$

We consider the constant $\mu^0 \le 1$ dependent on the degree of gas dissociation during passage through the shock to be known. Substituting (2.1), (2.2) into (1.1)-(1.3), we have the following system of equations in a first approximation (superscripts omitted):

$$u_t + u_x + vu_y + wu_z = 0; \quad (2.3)$$

$$\rho_t + \rho_x + (\rho v)_y + (\rho w)_z = 0; \quad (2.4)$$

$$w_t + w_x + vw_y + ww_z = 0; \quad (2.5)$$

$$\rho(v_t + v_x + vv_y + wv_z) = -p_y; \quad (2.6)$$

$$\rho(h_t + h_x + vh_y + wh_z) = -W_r B T^n; \quad (2.7)$$

$$h = \rho^{-1},\ \rho = \mu T^{-1},\ B = T^4. \quad (2.8)$$

For y = S*(x, z, t) on the shock we obtain the relationships

$$u_s = -S_x^*,\quad v_s = S_x^* + S_t^* - S_z^{*2} - 1,\quad w_s = -S_z^*,$$
$$p_s = 2(S_x^* + S_t^*) - S_z^{*2} - 1,\quad \rho_s = 1,\quad h_s = 1,\quad T_s = \mu. \quad (2.9)$$

On the wing surface for y = F*(x, z, t)

$$v_b = F_x^* + F_t^* + w_b F_z^*. \quad (2.10)$$

Here the dimensionless parameter

$$W_r = \frac{8\sigma T_\infty^4}{\rho_\infty V_\infty^3 \sin^3\alpha} r A^4$$

is the ratio between the radiant energy flux and the convective energy flux. Let the relationship of the small parameters ε, A^{-1}, be such that $W_r = O(1)$, i.e., $A = O[\varepsilon^{-1/(n+4)}]$. Therefore, in addition to the known geometric similarity parameter $\Omega = s/(L\varepsilon^{1/2}\tan\alpha)$ [4] (s is the wing semispan), the solution for radiating equilibrium gas flow around a wing will depend also on the parameters W_r and μ. The hypersonic law of plane sections for slender bodies at high angles of attack [5, 6] with the addition of these similarity parameters will be valid even for radiating gas flow around a wing since (2.4)-(2.8) do not contain perturbations of the longitudinal velocity u which is determined from (2.3) after (2.4)-(2.8) has been solved.

In conformity with the general result [7-9], a fundamental property of high-density radiating gas flow is conservation of the ratio between the vorticity flow component $\omega_v = w_y$ and the gas density along the particle motion trajectory in the shock layer. This property is expressed mathematically in the form of an integrable combination of (2.4) and (2.5):

$$(w_y/\rho)_t + (w_y/\rho)_x + v(w_y/\rho)_y + w(w_y/\rho)_z = 0. \quad (2.11)$$

This general conservation property [7-9] for high-density gas flow permits obtaining an interesting result of practical importance in the case of radiating gas flow. On the basis of the relationships (2.11) and (2.8), we arrive at the deduction that the product $\omega_v^4 B$ is conserved constant along the particle trajectory (along the streamline in the stationary case)

$$(\omega_v^4 B)_t + (\omega_v^4 B)_x + v(\omega_v^4 B)_y + w(\omega_v^4 B)_z = 0. \quad (2.12)$$

Therefore, there is a relationship between the vorticity flow component, which is a kinematic characteristic of the flow field, and the radiation intensity distribution, meaning the radiation thermal flux to the body. The relationship (2.12) set-up shows that the radiation intensity is greater in domains of weakly vortical flow than in strongly vortical flow domains, where this dependence is sufficiently strong since B is proportional to ω_v^{-4} (2.12).

3. Equation (2.11) is important not only from the physical but also from the mathematical viewpoint. The existence of a general integral for the nonlinear system of partial differential equations (2.4)-(2.8), that follows from (2.11), permits obtaining an analytic solution of this system. In fact, we use (2.11) together with (2.4) and we turn to the independent variables x, w, z, $\tau = x - t$. In the new variables, the system (2.11), (2.5)-(2.8) and the boundary conditions (2.9), (2.10) describing the nonstationary radiating gas flow in the shock layer acquire the same form as for the stationary flow, and contain functions dependent on τ as on a parameter:

$$(\rho y_w)_x + w(\rho y_w)_z = 0; \quad (3.1)$$

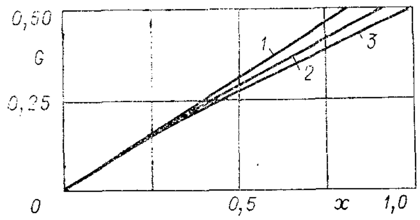

Fig. 2

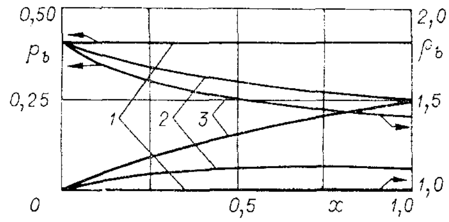

Fig. 3

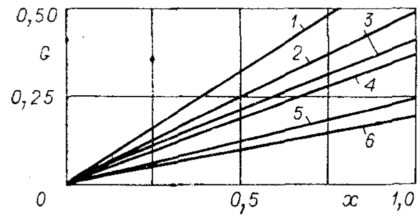

Fig. 4

$$v = y_x + wy_z; \tag{3.2}$$

$$p_w = -\rho y_w(v_x + wv_z); \tag{3.3}$$

$$\rho^{n+3}(\rho_x + w\rho_z) = K/(n+4); \tag{3.4}$$

for $y = S(x, z, \tau) = S^*(x, z, x - \tau)$ or $w = -S_z(x, z, \tau)$

$$v_s = S_x - S_z^2 - 1, \quad p_s = 2S_x - S_z^2 - 1, \quad \rho_s = 1; \tag{3.5}$$

for $y = F(x, z, \tau) = F^*(x, z, x - \tau)$ or $w = w_b(x, z, \tau)$

$$v_b = F_x + w_b F_z. \tag{3.6}$$

The parameters W_r, μ are in the problem for y, v, p, ρ, h only in the form of the combinations $K = W_r\mu^{n+4}(n + 4)$. Double integration of (3.1) yields ($\Gamma = \omega_v^{-1}\rho$ is an arbitrary function defined below)

$$y = F(x, z, \tau) + \int_{w_b}^{w} \frac{\Gamma(w', z - w'x, \tau)}{\rho(x, w', z, \tau)}\, dw'. \tag{3.7}$$

Let us note that in addition to w the quantities $\theta = z - wx$, τ are constant along the trajectories as can be seen by evaluating the total derivatives of these quantities as had been done for w in (2.5). Using (3.7), we obtain a formula for v from (3.2)

$$v = F_x + wF_z + \int_{w_b}^{w} \left[(w - w')\,\Gamma_{\theta'} - \frac{\Gamma}{\rho}(\rho_x + w\rho_z)\right]\frac{dw'}{\rho} - \frac{\Gamma_b}{\rho_b}[(w_b)_x + w(w_b)_z], \tag{3.8}$$

which satisfies condition (3.6) for the satisfaction of one of the relationships

$$\Gamma_b = 0; \tag{3.9}$$

$$(w_b)_x + w_b(w_b)_z = 0. \tag{3.10}$$

The pressure distribution is found from (3.3), (3.7), (3.8) in the form

$$p = p_s + \int_{w}^{-S_z} (v_x + w'v_z)\,\Gamma(w', z - w'x, \tau)\, dw'. \tag{3.11}$$

To determine the form of the function Γ we use (3.7) and (3.8) for $w = w_s$ and the conditions (3.5) for v_s, ρ_s. Since $\rho_s = 1$, we consequently obtain analogously to [7]

$$\Gamma_s(x, z, \tau) = (S_z S_{zz} - S_{zx})^{-1}. \tag{3.12}$$

Let $\chi(w, \theta, \tau)$ denote the abscissa of the intersection between a given gas particle trajectory and the shock defined as the root of the functional equation

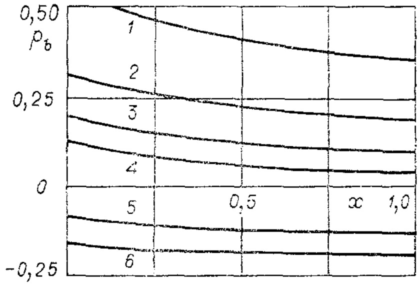

Fig. 5

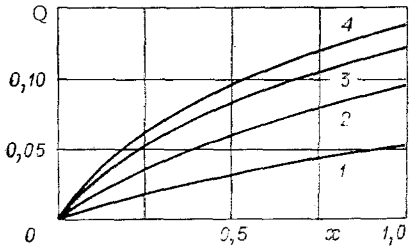

Fig. 6

$$w = -S_z(\chi, w\chi + \theta, \tau). \tag{3.13}$$

Then the function Γ is expressed in the flow field in the form

$$\Gamma(w, \theta, \tau) = \Gamma_s(\chi, w\chi + \theta, \tau). \tag{3.14}$$

Equation (3.4) in the variables x, w, θ, τ has the form

$$\rho^{n+3}\rho_x = K/(n+4).$$

Integrating this with the boundary condition $\rho_s = 1$ for $x = \chi(w, \theta, \tau)$ taken into account, we obtain the density distribution

$$\rho(x, w, \theta, \tau) = \{1 + K[x - \chi(w, \theta, \tau)]\}^{1/(n+4)} \tag{3.15}$$

and the values of the temperature and enthalpy

$$T = \mu h = \mu\{1 + K[x - \chi(w, \theta, \tau)]\}^{-1/(n+4)}.$$

from (2.8). Hence, it is seen that in the case of an optically transparent gas layer, radiation results in a diminution in the temperature and enthalpy and an increase in the density along the particle motion trajectories.

Therefore, the gasdynamic and thermodynamic radiation gas functions are expressed in the form of quadratures and functional relations in terms of the shape of the shock surface which, according to (3.7), (3.12), (3.15), should be determined jointly with the function Γ from the system of two equations

$$S(x, z, \tau) = F(x, z, \tau) + \int_{w_b}^{-S_z} \frac{\Gamma(w, z - wx, \tau)\, dw}{\{1 + K[x - \chi(w, z - wx, \tau)]\}^{\frac{1}{n+4}}}, \tag{3.16}$$
$$\Gamma(-S_z, z + S_z x, \tau) = (S_z S_{zz} - S_{zx})^{-1}.$$

Let us note that the function $w_b(x, z, \tau)$ is defined analogously [10].

If the approximation of an optically transparent gas layer, and therefore, of (3.15) are not applicable, then the analytic expression for the density can be found either from the solution of other approximate equations or by approximating the results of a numerical computation of one-dimensional radiating gas flow at constant pressure. The fundamental formulas (3.7)-(3.12) remain valid here.

4. Extending the results obtained in the stationary case [8], we convert the solution obtained by taking χ as a new independent variable in place of w. We have along the trajectory of this particle

$$z - \zeta = -S_z(\chi, \zeta, \tau)(x - \chi), \quad w = -S_z(\chi, \zeta, \tau), \tag{4.1}$$

where ζ is the z coordinate of the intersection between this trajectory and the shock. Differentiating these equalities for constant x, z, τ with (3.12) and (3.14) taken into account, we obtain

$$dw = d\chi/\{\Gamma[1 - (x - \chi)S_{zz}(\chi, \zeta, \tau)]\}.$$

We now obtain in place of the system (3.16) for the shock shape, an equation

$$S(x, z, \tau) = F(x, z, \tau) + \int_{\chi_b}^{x} \frac{[1 + K(x-\chi)]^{-\frac{1}{n+4}}}{1-(x-\chi) S_{zz}(\chi, \zeta, \tau)} d\chi, \tag{4.2}$$

which should be solved in conjunction with (4.1). The expressions for the vertical velocity component and the pressure acquire the form

$$v = F_x - S_z(\chi, \zeta, \tau) F_z + \int_{\chi_b}^{\chi} \left\{ \frac{[1-(x-\chi') S'_{zz}] S'_{zz} + (x-\chi')(S'_z - S_z) S'_{zzz}}{[1-(x-\chi') S'_{zz}]^3 [1+K(x-\chi')]^{\frac{1}{n+4}}} - \right. \tag{4.3}$$

$$\left. - \frac{K[1+K(x-\chi')]^{-\frac{n+5}{n+4}}}{(n+4)[1-(x-\chi') S'_{zz}]} \right\} d\chi' - \frac{(\chi_b)_x - S_z(\chi_b)_z}{[1-(x-\chi_b) S_{zzb}][1+K(x-\chi_b)]^{\frac{1}{n+4}}};$$

$$p = 2S_x - S_z^2 - 1 + \int_{\chi}^{x} \frac{v_x - S'_z v_z}{1-(x-\chi') S'_{zz}} d\chi', \tag{4.4}$$

where $S'_z = S_z(\chi', \zeta', \tau)$, etc., and $S_{zzb} = S_{zz}[\chi_b, z_e(\chi_b), \tau]$.

Since the shock layer can be considered locally one-dimensional in the problem under consideration when radiation transfer is taken into account, we used a one-dimensional approximation analogous to [2] to calculate the radiation heat flux. Then applying the known solution [11] of the radiation transport equation and neglecting radiation from the wing surface, we obtain the following expression for the magnitude of the local radiation heat flux to the wing:

$$q_b(x, z) = \frac{\rho_\infty V_\infty^3}{2} \sin^3 \alpha Q(x, z), \tag{4.5}$$

$$Q(x, z) = \frac{K}{2(n+4)} \int_{\chi_b}^{x} \frac{[1+K(x-\chi)]^{-\frac{n+5}{n+4}}}{1-(x-\chi) S_{zz}(\chi, \zeta, \tau)} d\chi.$$

In writing (4.2)-(4.5) it is taken into account that $\chi = x$ on the shock. The values of χ_b for trajectories lying on the wing surface will be found by assuming the shape of the wing leading edge to be independent of the time in planform $z = z_e(x)$.

If the shock is attached to the edge, i.e., $S_e(x, \tau) = F_e(x, \tau)$ along it, then by satisfying (3.5) by using (3.6), we obtain on the edge

$$w_e(x, \tau) = -S_z^e = \frac{1}{2}\left[z'_e - F_z^e - \sqrt{(z'_e + F_z^e)^2 - 4}\right], \tag{4.6}$$

where $S_e(x, \tau) = S[x, z_e(x), \tau]$; $F_e(x, \tau) = F[x, z_e(x), \tau]$; $F_z^e = F_z[x, z(x), \tau]$. To realize such a regime, it is therefore required that $|z_e + F_z^e| \geq 2$. For the flow around a wing with a smooth edge shape in planform, χ_b is determined from (4.1) in which $\zeta = z_e(\chi)$, $S_z = S_z^e$ should be put. We represent the shape of an edge having a break at $x = 0$ in the form of two smooth pieces $z = z_e(x)$ for $x \geq 0$ and $z = -z_e(x)$ for $x \leq 0$ $(z'_e(0) \neq 0)$. Then for $|z| \leq w_e(+0, \tau)x$ in the central part of the wind, there is a fan of trajectories passing through the apex. Therefore $\chi_b = 0$ for them. Between the outer trajectories of this fan and the leading edge, χ_b is determined from (4.1) and (4.6). This case is illustrated in Fig. 1.

If the shock is attached just to the apex and is detached from the edge, then all trajectories on the wing pass through the apex and $\chi_b = 0$ on the whole wing surface.

5. Because of the presence of functional relations, the solution of the system (4.1) is fraught with known mathematical difficulties in the general case. However, a class of its particular exact solutions exists which corresponds to the following body surface and shock types:

$$F(x, z, \tau) = -\frac{f(x, \tau) z^2}{2}, \quad S(x, z, \tau) = G(x, \tau) - \frac{f(x, \tau) z^2}{2}, \quad \chi_b = 0. \tag{5.1}$$

The shock layer thickness equals

$$G(x,\tau)=\int_0^x \frac{[1+K(x-\chi)]^{-\frac{1}{n+4}}}{1+(x-\chi)f(\chi,\tau)}\,d\chi.$$

The case $f(x, \tau) = b/x$ corresponds to the stationary flow around a conical wing of invariant shape with transverse curvature equal to $b/(x \tan\alpha)$ in the plane of symmetry ($z = 0$). The shock layer thickness is constant here in the transverse section x = const

$$G(x)=\int_0^x \frac{[1+K(x-\chi)]^{-\frac{1}{n+4}}}{(1-b)\chi+bx}\,\chi d\chi. \tag{5.2}$$

For small K we obtain the following approximate formula from (5.2)

$$G(x)\simeq x\eta_{s0}-\frac{Kx^2}{(n+4)(1-b)}\left(\eta_{s0}-\frac{1}{2}\right),\quad \eta_{s0}=\frac{b\ln b+1-b}{(1-b)^2}.$$

In the Newtonian approximation the radiation does not influence the pressure distribution (see [2], for example). The solution of the problem in a first approximation to the Newtonian solution permits taking account of the influence of the radiation on the pressure. The calculation of the pressure distribution along the line of intersection of the plane of symmetry with the wing surface by means of (5.1), (5.2), (4.3), (4.4) results in the equation

$$p_b(x)=2G'(x)-1+\int_0^x\left\{\frac{2b^2}{[(1-b)\chi+bx]^2}+\frac{2b\rho_x}{\rho[(1-b)\chi+bx]}+\right.$$
$$\left.+\frac{2\rho_x^2}{\rho^2}-\frac{\rho_{xx}}{\rho}\right\}\left[\frac{x-\chi}{1-b}-\frac{bx}{(1-b)^2}\ln\frac{x}{(1-b)\chi+bx}\right]\frac{\chi d\chi}{\rho[(1-b)\chi+bx]}. \tag{5.3}$$

According to (4.5) and (5.1), the radiation heat flux is constant along the span, which is a corollary of the constancy of the compression layer thickness

$$Q(x)=\frac{K}{2(n+4)}\int_0^x \frac{[1+K(x-\chi)]^{-\frac{n+5}{n+4}}}{(1-b)\chi+bx}\,\chi d\chi.$$

To obtain the upper bound of the influence of radiation we later set $n = 0$. Results of computations by means of (5.2) and (5.3) are represented in Figs. 2 and 3 for $b = 0.5$ and $K = 0, 2, 4$ (curves 1-3, respectively). It is seen that the radiation results in a diminution in the shock layer thickness, in longitudinal curvature of the compression shock, and also to a pressure drop along the root chord of the wing as compared with a flow without radiation ($K = 0$). The density distribution $\rho_b(x)$ along the root chord, is also given in Fig. 3, as obtained from (3.15) for $\chi = 0$. The influence of the curvature parameter b on the shock layer thickness and the pressure distribution is illustrated in Figs. 4 and 5 for $K = 2$ (curves 1-6 correspond to b = 0.25, 0.75, 1.25, 1.75, 4.75, 7). As the parameter b grows, the correction to the Newtonian value of the pressure changes sign and becomes negative (Fig. 5), while the pressure drop between the apex and the take of the wing, which characterizes the influence of the radiation, diminishes.

The results obtained show that in the flow around a conical wing, the shape of the compression shock and the whole flow do not, in contrast to [4], possess the property of conicity because of the influence of radiation, and should be investigated on the basis of three-dimensional flow theory.

The radiation heat flux distribution along the root chord of the wing is shown in Fig. 6 for $b = 0.25$ and K = 0.8, 2, 3.2, 4 (curves 1-4, respectively). As K increases and b = const, the shock layer thickness diminishes somewhat (see Fig. 2) while the radiation intensity grows strongly, which results in growth of the heat flux. For fixed K the shock layer thickness diminishes as b grows (see Fig. 4), whereupon the heat flux to the wing becomes less.

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EQUATIONS OF MECHANICS OF A GAS-PARTICLE MIXTURE

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We consider the non-steady, one-dimensional motion of a gas containing suspended particles. For subsonic relative velocities of the gas and particles, the equations of the system have two complex characteristics [1] corresponding to instability of the solution to the Cauchy problem. The physical cause of the instability [2, 3] is a rise in the filtration velocity of the gas and a corresponding drop in pressure in regions where there is an increase in particle concentration. The pressure gradient encourages particle coagulation and perturbations grow exponentially. The rate of growth is inversely proportional to the wavelength of the perturbation.

It is important to be able to separate real physical flow instabilities from formal instability arising because of approximations in describing the mixture. An example of the latter is the rapid growth of short wavelength perturbations. In [3] an essential difference was pointed out between problems admitting steady motion of the phases (suspension of layer, precipitation of a suspension) from those of the non-steady type (passing of a shock wave through a suspension in gas). In the latter case, the velocity of relative motion of the phases goes to zero with time, and if the nonphysical fluctuations are removed, the Cauchy conditions can be correct. In the numerical solution of such problems, this is always understood.

In [3, 4] the random motion of the particles was considered as a stabilizing effect. In the present paper, we consider the non-steady-state problem at small particle concentrations, where the random motion of particles is not important. The equations obtained here include explicitly the interphase forces and the relative volume of the dispersed phase averaged over the volume of the particle. Thus the growth of short wavelength perturbations is suppressed.

1. Statement of the Problem and Preliminary Estimates. Following the treatment in [1, 3], we ignore the internal properties of the subsystem in the equations of mass and momentum and limit the discussion (as in [3]) to the case of a barotropic gas. Thus we do not have to deal with the energy equation. The system of equations has the form [3, 5]:

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